Below are a series of questions – slightly different than ones you've seen before. However, you have the capabilities to solve all of them. My estimation of their challenge level is designated with (*) (**) or (***), from slightly challenging to very challenging. *I want you to pick the problems that seem the most interesting to you*.

You may go it alone, or you may work with a single partner. If you work alone, you have to do enough problems so that you have at least *** asterisks… So you could turn in A, B, and C. Or you could just do G. Or you could do A and H.

If you work with a partner, you must have at least **** asterisks. You should work together collaboratively – meaning you are working *together*. You should each be working on all the problems, and each doing your own writing. Please staple your papers together.

YOU MAY ONLY CONSULT ME, YOUR NOTES, CALCULATOR, AND (if applicable) YOUR PARTNER

You will be graded out of 10 spot check points, and will involve your ability to solve the problems AND explain your solutions. Your solutions should be clear – and be "teaching" someone else how to solve the problem. This means you want to do a lot of writing – explaining each step you take and why you take it. You ought to show me your work before you turn in your final draft – so I can make comments and help you out. You can also come to me if you are getting stuck. I won't tell you how to solve the problem, but I may be able to prompt you along.

A (*). Determine coefficients a, b, c, and d such that $f(x)=ax^3+bx^2+cx+d$ satisfies $f(1)=1$, $f(-1)=5$, $f'(2)=2$, and $f''(-1)=3$.

B (*). Where (at which x values) do the functions $f(x)=2x^2+5x-3$ and $g(x)=5x^2-2x-13$ have perpendicular tangent lines?

C (*). Part I. If $f'(x)=\frac{1}{\sqrt{x}}+4x+15$, can you find what $f(x)$ is? In fact, there are multiple answers for $f(x)$! Can you find another one?

Part II. If $f''(x)=e^x(x^2+4x+2)$, can you find what $f(x)$ is? In fact, there are multiple answers for $f(x)$! Can you find another one?

D (*). Part I: Define what it should mean to say that two curves intersect at right angles.

Part II: Prove that the curves $y=\frac{1}{x}$ and $y=\frac{1}{2-x}$ intersect at right angles.

E (*). Calculate the first four derivatives of $f(x)=x^2e^x$. (So calculate $f'(x)$, $f''(x)$, $f'''(x)$, $f''''(x)$.) Then come up with the formula for the *n*th derivative of $f(x)$.

F (**). Part I. Carefully sketch the graph of $f(x)=x^2-5x+4$ and $g(x)=-2x+3$. Find the value(s) of x at which the graphs have parallel tangent lines.

Part II. Using your graphing calculators, graph $f(x)=3x^2-5x+1$ and $g(x)=x^3-2x^2+x-3$. Find the value(s) of x at which the graphs have parallel tangent lines.

G (***). Dos Moscas[1]

Two flies (twin sisters) are sitting on a spherical balloon while it is being inflated at a constant rate. Assume that air is being injected into the balloon at a rate of 5 cubic centimeters per second, and that the balloon has no air in it to begin with. Further assume that one sister is situated at the north pole and the other on the equator.

a. Draw a picture complete with labels.

b. How fast are the two flies parting company as a function of time?

c. How fast after 3 seconds?

d. How fast after 1/10 second?

e. How fast after 1/100 second?

f. How fast are they separating initially?

Hint: This problem has a couple twists. Write down all your knowns and unknowns – and see if you can work with them to find whatever variables you are missing!

H (**). Part I: Find the equation of the tangent line to the hyperbola $xy=1$ (alternatively written $y=\frac{1}{x}$) at $x=2$. Sketch the hyperbola and this tangent line. Find the area of the triangle bounded by the tangent line, the x-axis, and the y-axis.

Part II: Find the equation of the tangent line to the hyperbola $xy=1$ (alternatively written $y=\frac{1}{x}$) at $x=3$. Sketch the hyperbola and this tangent line. Find the area of the triangle bounded by the tangent line, the x-axis, and the y-axis.

Part III. Find the equation of the tangent line to the hyperbola $xy=1$ (alternatively written $y=\frac{1}{x}$) at $x=a$. Find the area of the triangle bounded by the tangent line, the x-axis, and the y-axis.

[1] Marcus Cohen, et al., *Student Research Projects in Calculus* (The Mathematical Association of America: Washington DC, 1991): 101