A Strategy for Solving Related Rates Problems

Step 1: Assign letters to all quantities that vary with time and any others that seem relevant to the problem. Give a definition for each letter.

Step 2: Identify the rates of change that are known and the rates of change that is to be found. Interpret each rate as a derivative.

Step 3: Find an equation that relates the variables whose rates of change were identified in Step 2. To do this, it will often help to draw an appropriately labeled figure to illustrate the relationship.

Step 4: Differentiate both sides of the equation obtained in Step 3 with respect to time to produce a relationship between the known rates of change and the unknown rate of change.

Step 5: *After* completing Step 4, substitute all known values for the rates of change and the variables, and then solve for the unknown rate of change.

CLASS PROBLEM 1

Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 5 ft/s. How fast is the area of the spill increasing when the radius of the spill is 25 ft?

Step 1: Variables and what they mean

Step 2: Rates of change (known *and* unknown)

Step 3: Draw a picture. Find an equation relating the variables in Step 1.

Step 4: Differentiate the equation from the previous step with respect to time

Step 5: Substitute in values and solve for the unknown rate of change

CLASS PROBLEM 2

Let A be the area of a square whose sides have length s and assume that s varies with the time t. At a certain instant the sides are 3 ft long and increasing at a rate of 2 ft/min. How fast is the area increasing at that moment?

Step 1: Variables and what they mean

Step 2: Rates of change (known *and* unknown)

Step 3: Draw a picture. Find an equation relating the variables in Step 1.

Step 4: Differentiate the equation from the previous step with respect to time

Step 5: Substitute in values and solve for the unknown rate of change

PARTNER PROBLEM 1

Let l be the length of a diagonal of a rectangle whose sides have lengths x and y, and assume that x and y vary with time. If x increases at a constant rate of 1/2 ft/sec and y decreases at a constant rate of 1/4 ft/sec, how fast is the size of the diagonal changing when x=3 ft and y=4 ft? **[Hint: Pay attention to the words "increasing" and "decreasing"]**

Step 1: Variables and what they mean

Step 2: Rates of change (known *and* unknown)

Step 3: Draw a picture. Find an equation relating the variables in Step 1.

Step 4: Differentiate the equation from the previous step with respect to time

Step 5: Substitute in values and solve for the unknown rate of change

PARTNER PROBLEM 2

Let θ (in radians) be an acute angle in a right triangle, and let *x* and *y*, respectively, be the lengths of the sides adjacent to and opposite θ. Suppose also that *x* and *y* vary with time.

At a certain instant, *x*=2 units and is increasing at 1 unit/sec, while *y*=2 units and is decreasing at 1/4 unit/sec. How fast is θ changing at that instant? **[Hint: Pay attention to the words "increasing" and "decreasing"]**

Step 1: Variables and what they mean

Step 2: Rates of change (known *and* unknown)

Step 3: Draw a picture. Find an equation relating the variables in Step 1.

Step 4: Differentiate the equation from the previous step with respect to time

Step 5: Substitute in values and solve for the unknown rate of change

INDIVIDUAL PROBLEM 1

A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/sec. How rapidly is the area enclosed by the ripple increasing at the end of 10 seconds?

Step 1: Variables and what they mean

Step 2: Rates of change (known *and* unknown)

Step 3: Draw a picture. Find an equation relating the variables in Step 1.

Step 4: Differentiate the equation from the previous step with respect to time

Step 5: Substitute in values and solve for the unknown rate of change

INDIVIDUAL PROBLEM 2

A 13 ft ladder is leaning against a wall. If at a certain instant the bottom of the ladder is 2 ft from the wall and is being pushed toward the wall at the rate of 6 in/sec, how fast is the acute angle that the ladder makes with the ground increasing? [**Hint: Pay attention to the units in this problem!**]

Step 1: Variables and what they mean

Step 2: Rates of change (known *and* unknown)

Step 3: Draw a picture. Find an equation relating the variables in Step 1.

Step 4: Differentiate the equation from the previous step with respect to time

Step 5: Substitute in values and solve for the unknown rate of change

INDIVIDUAL PROBLEM 3

A spherical balloon is inflated so that its volume is increasing at the rate of 3 ft^3/min. How fast is the radius of the balloon increasing when the radius is 1 ft? [Recall the volume of a sphere is: $V = \frac{4}{3}\pi r^3$]

Step 1: Variables and what they mean

Step 2: Rates of change (known *and* unknown)

Step 3: Draw a picture. Find an equation relating the variables in Step 1.

Step 4: Differentiate the equation from the previous step with respect to time

Step 5: Substitute in values and solve for the unknown rate of change